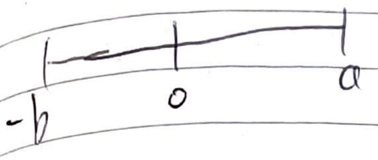
Taylor Expansion (Belyaev 1999)

No.
Date. 3[redacted] · 7 · 2017



$f(0) \equiv f$, $f(a) \equiv f_a$, $f(-b) \equiv f_{-b}$

$$f(a) = f + af' + \frac{a^2}{2}f'' + \dots$$

$$f(-b) = f - bf' + \frac{b^2}{2}f'' + \dots$$

$$f' = \frac{f_a - f}{a} - \frac{a}{2}f'' + \dots$$

$$f' = \frac{f_{-b} - f}{-b} + \frac{b}{2}f'' + \dots$$

$$(a+b)f' \approx \frac{b}{a}(f_a - f) - \frac{a}{b}(f_{-b} - f)$$

$$= \frac{a^2 - b^2}{ab}f + \frac{b^2 f_a - a^2 f_{-b}}{ab}$$

$$\therefore f' = \frac{a-b}{ab}f + \frac{(b^2 - a^2)f_a + a^2 f_a}{ab(a+b)} + \frac{(b^2 - a^2)f_{-b} - b^2 f_{-b}}{ab(a+b)}$$

$$= \frac{f}{a} - \frac{f}{b} + \frac{b-a}{ab}f_a + \frac{a}{b}\frac{f_a}{a+b} + \frac{b-a}{ab}f_{-b} - \frac{b}{a}\frac{f_{-b}}{a+b}$$

$$= \frac{f}{a} - \frac{f}{b} + \frac{f_a}{a} + \frac{-(a+b)f_a + af_a}{b(a+b)} - \frac{f_{-b}}{b} + \frac{(a+b)f_{-b} - bf_{-b}}{a(a+b)}$$

$$= \frac{f_a - f}{a} + \frac{f - f_{-b}}{b} - \frac{f_a - f_{-b}}{a+b}$$

$$f'' = \frac{2(f_a - f)}{a^2} - \frac{2}{a}f' + \dots$$

$$f'' = \frac{2(f_{-b} - f)}{b^2} + \frac{2}{b}f' + \dots$$

$$(a+b)f'' \approx 2\left[\frac{1}{a}(f_a - f) + \frac{1}{b}(f_{-b} - f)\right]$$

$$= \frac{2f_a}{a(a+b)} - \frac{f}{ab} + \frac{2f_{-b}}{b(a+b)}$$

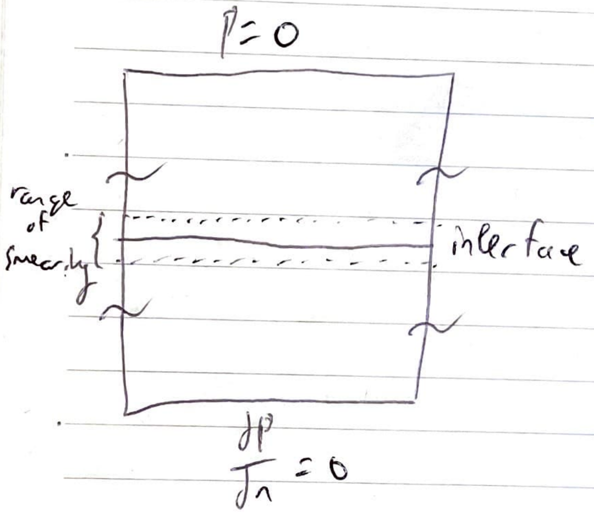
$$\nabla H = \delta = \nabla p$$
$$\nabla\cdot\delta = \nabla^2 p \Rightarrow p = H, \quad \nabla\times\delta = \nabla\times\nabla H = 0$$

From the previous calculation of curl of delta function, we know that $\delta \neq \nabla H$. Thus, we can decompose delta function into two components,

$$\delta = \nabla H + \varepsilon$$
$$0 \neq \nabla\times\delta = \nabla\times\nabla H + \nabla\times\varepsilon = \nabla\times\varepsilon$$

model problem : $\nabla^2 p = \nabla\cdot\delta = \nabla^2 H + \nabla\cdot\varepsilon$

$$\nabla^2 E = \nabla^2 (P-H) = \nabla\cdot\varepsilon$$



integration over the range of smearing

$$\int \nabla\times\delta\, dV = \int \nabla\times\varepsilon\, dV$$
$$= \oint n\times\varepsilon\, ds$$

$$\int |\nabla\times\delta|\, dV = \oint |n\times\varepsilon|\, ds$$

$$G\,\Delta V = |\varepsilon| L \qquad \Delta V = L\cdot h$$

$$\therefore |\varepsilon| = G h$$

No.
Date 17 · 8 · 2017

IBM Convergence

$\varepsilon \to 0$ as $h \to 0$

however, $\nabla^2 E = \nabla \cdot \varepsilon = \frac{\partial \varepsilon}{\partial n} = \text{constant}$ $\therefore |\varepsilon| \propto h$

$E \to 0$ as $h \to 0$